SHAPES, CURVES AND FONTS

2011 Introduction to Graphics

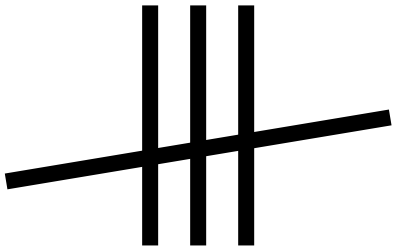
Lecture 6

Overview

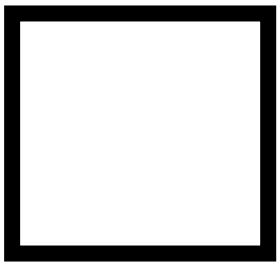
- Shapes
  - Identity parade
- Curves
  - Bezier curves
- Paths
  - Collections of curves
- Fonts
  - Essentially paths

Shape Type Review

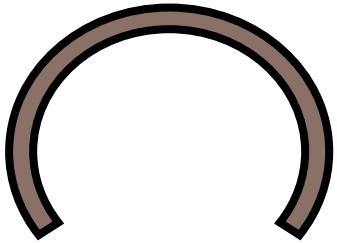
- Lines



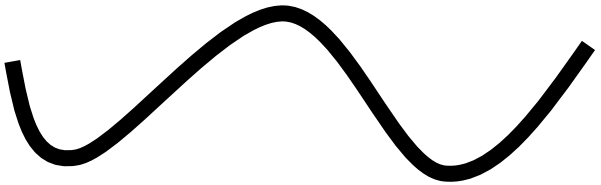
- Boxes



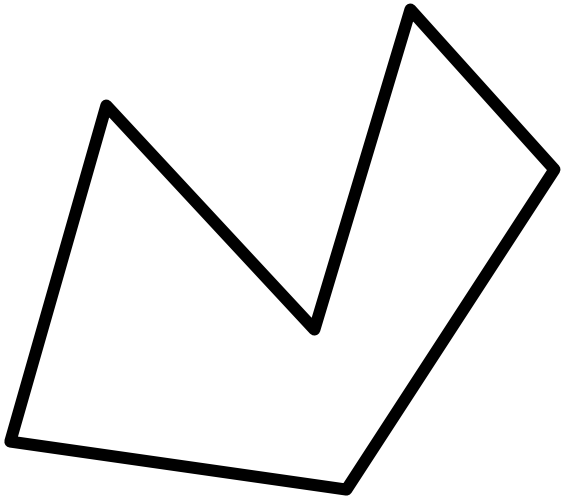
- Arcs



- Curves

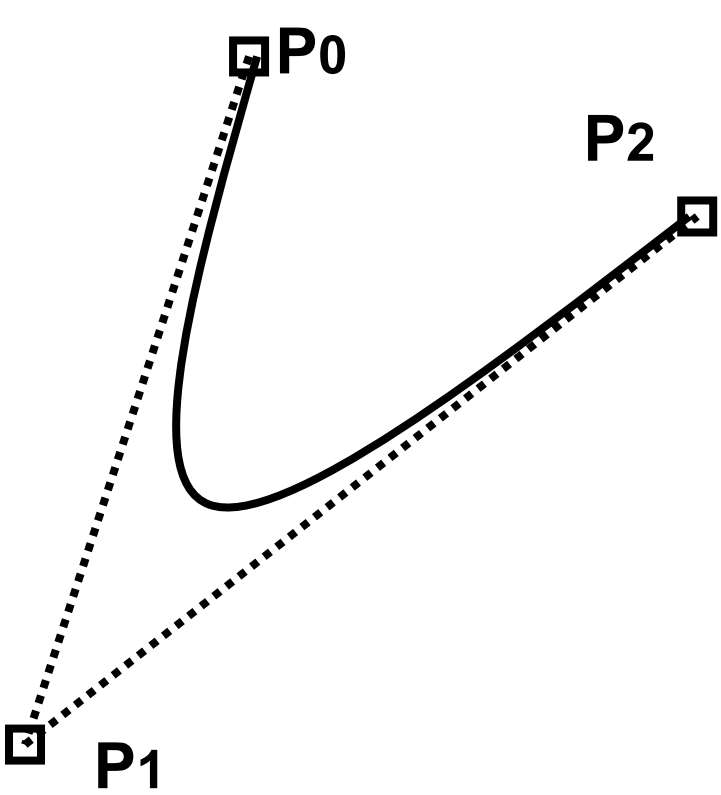


- Paths

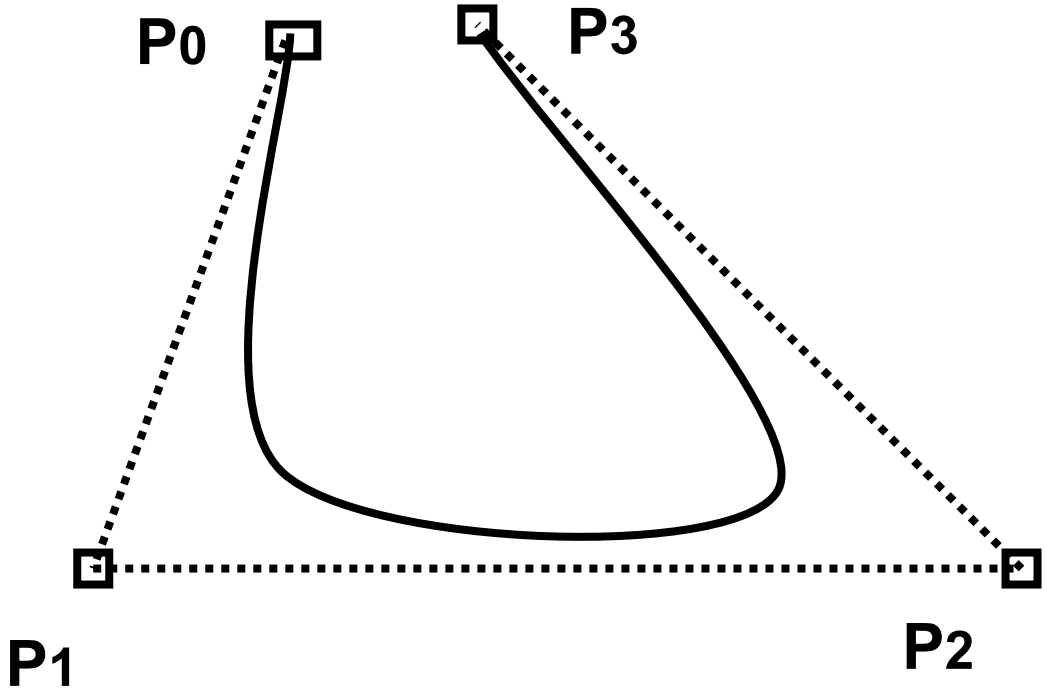


Curves

- Quad2D



- Cubic2D



Math Revision

- Explicit equation form $y = x^2$

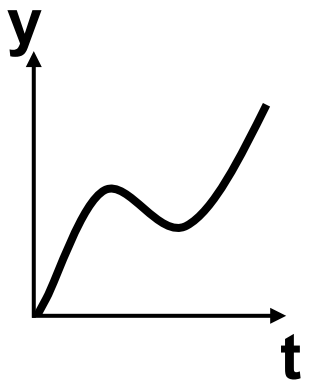
- Implicit $f(x, y) = 0$

- Parametric
$$x = r \bullet \cos(t)$$
$$y = r \bullet \sin(t)$$
$$t > 0, t < 2\Pi$$

Cubic Curve

- Many types of cubic equation
- General 1D:

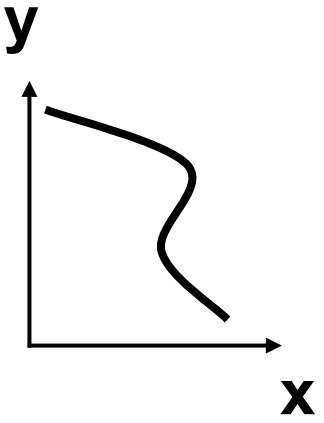
$$y(t) = a + bt + ct^2 + dt^3$$



- General 2D: one curve for each dimension

$$x(t) = a_x + b_x t + c_x t^2 + d_x t^3$$

$$y(t) = a_y + b_y t + c_y t^2 + d_y t^3$$



Cubic Curve

- General form is hard to edit
  - Parameters a,b,c,d have non-intuitive influence
  - Better to have **control points**
- Bézier curves are a classic example
  - Two end-points, two control points
  - Curve remains within convex hull of four points
  - User control is fairly intuitive

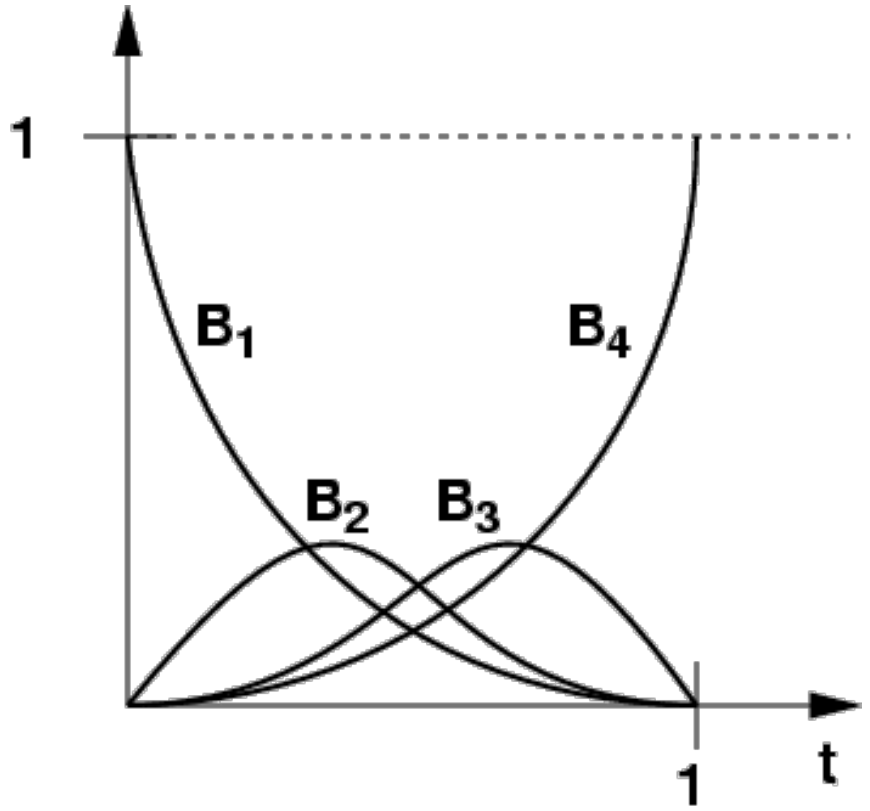
Form of the Equation

$$B(t) = (1-t)^3 P_0 + 3t(1-t)^2 P_1 + 3t^2(1-t)\ P_2 + t^3 P_3$$

- Note that intuitively this means that when t is near 0, the P_0 dominates.
  - The influence of each point is known as a weighting function
    - B_0 is the weight of P_0, B_1 is the weight of P_0 etc…

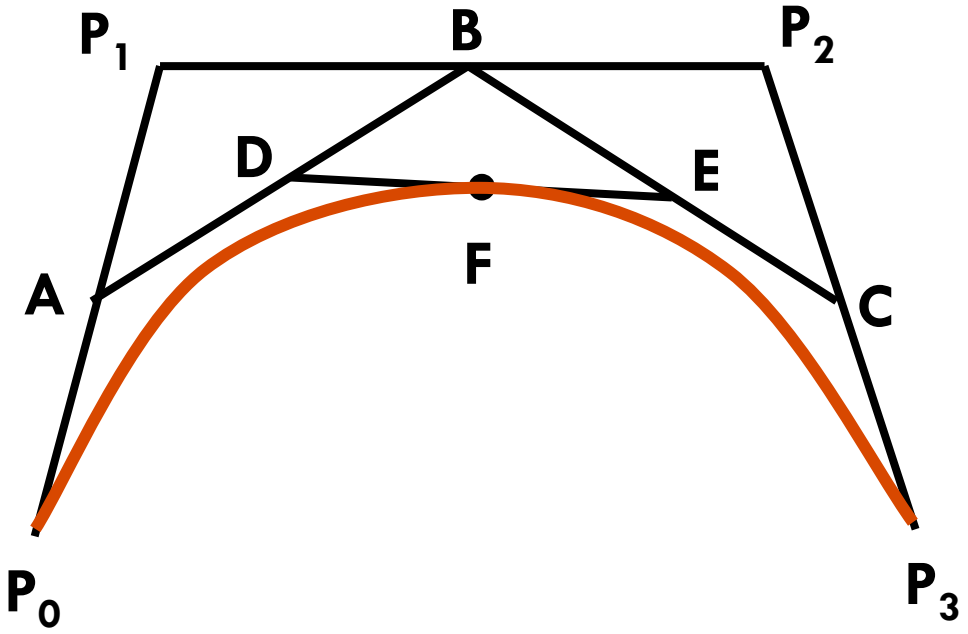
Bernstein Polynomials

- Plot of basis functions



Bisecting a Bézier

- Recursive split (de Casteljau's algo.)



If

A bisects $\mathbf{P_0P_1}$

B bisects $\mathbf{P_1P_2}$

C bisects $\mathbf{P_2P_3}$

D bisects **AB**

E bisects **BC**

F bisects **DE**

Then

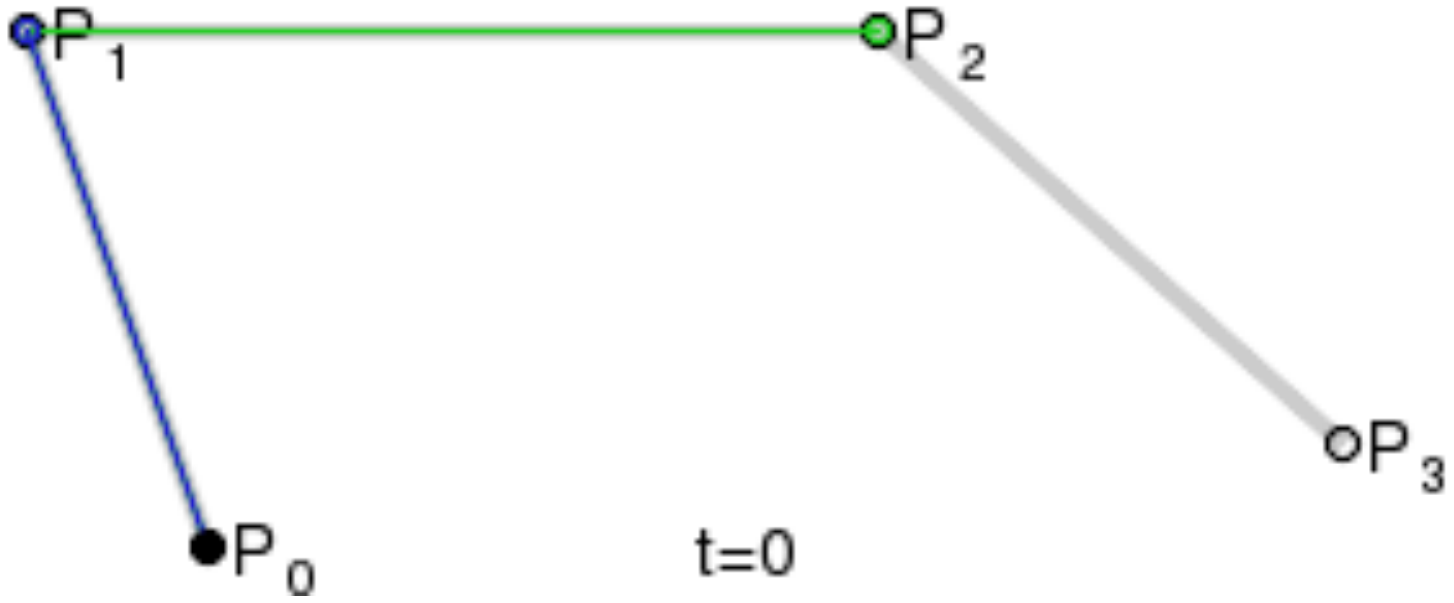
$\mathbf{P_0ADF}$ form new Bézier

$\mathbf{FECP_3}$ form new Bézier

Drawing a Bézier

- The Bézier bisection process suggest the following drawing process:
  1. If P_0P_1 P_2P_3 are "close" together
     - Close might mean, within 2 pixels of each other other the screen
  2. Then plot the straight line P_0P_3
  3. Else bisect P_0P_1 P_2P_3
  4. Recurse to step 1 on both sub-curves

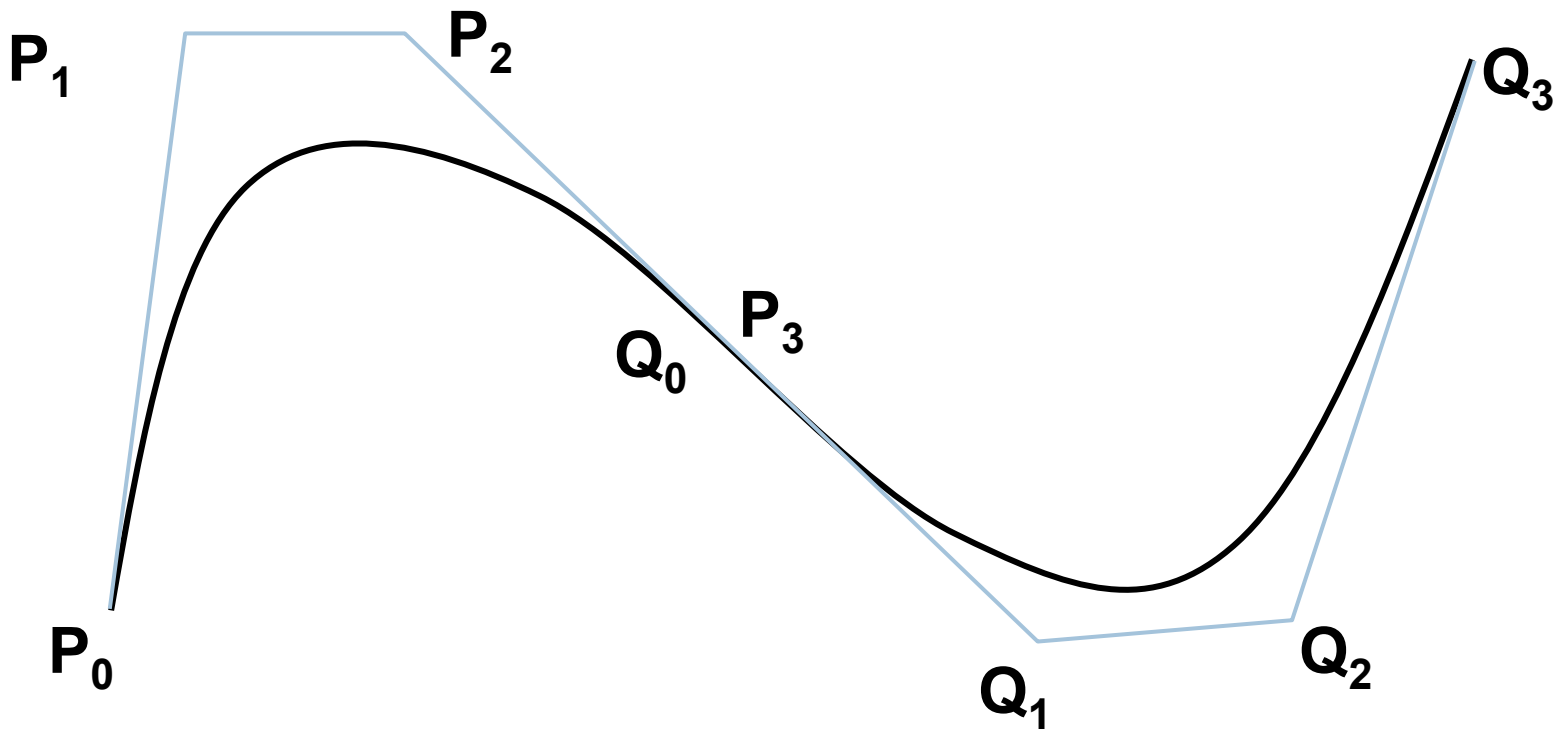
Another View of a Cubic Bézier



Smoothness

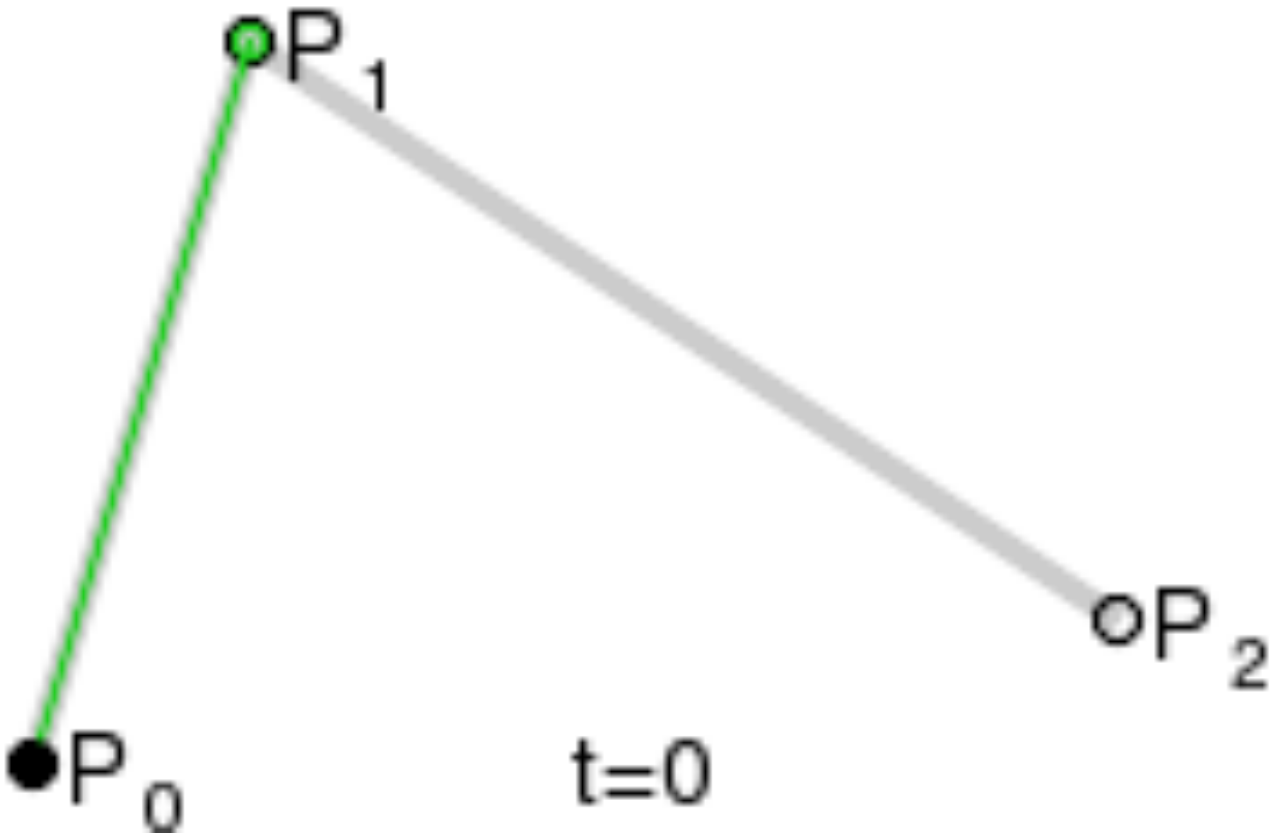
- Note that the line P_0P_1 defines a tangent to the curve at P_0
- Common requirement to join two bezier curves together ($P_{0\text{-}3}$, $Q_{0\text{-}3}$)
- This requires:
  - The points P_3 equals Q_0
  - Tangents to be equal
    - I.e., P3 (Q0), P2, Q1 are **collinear**
  - Called C_1 continuity (1st derivative is continuous)
  - C_0: only positions are continuous (i.e. $P_3 = Q_0$)

Joining Bézier curves



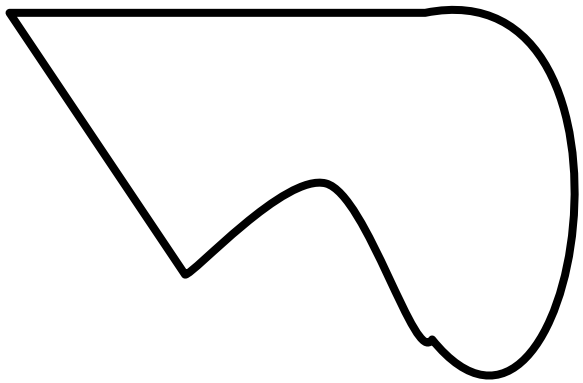
Asymmetric: Curve goes through some control points but misses others

Quadric Bézier Example



Complex Shapes

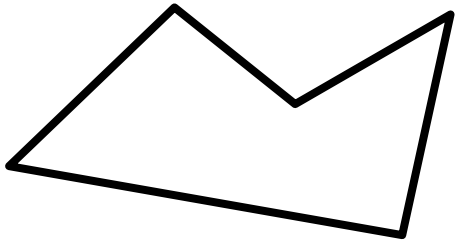
- Paths
  - A **path** is a list of **segments** where each segment is a line, quad or cubic

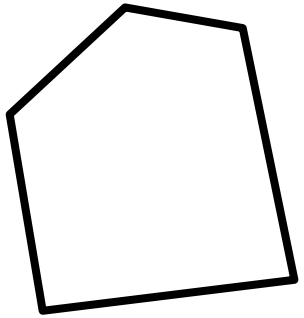


  - Edge are ordered, usually anti-clockwise

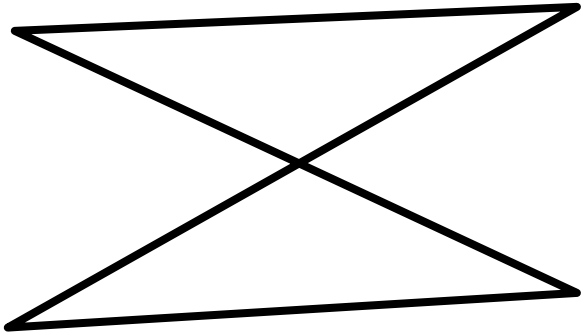
Types of Shape

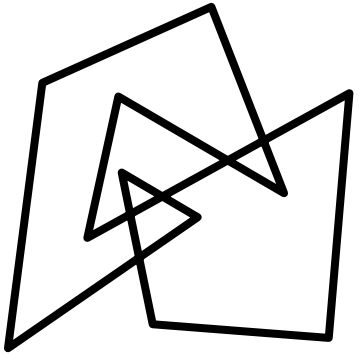
- Simple - Concave, Convex





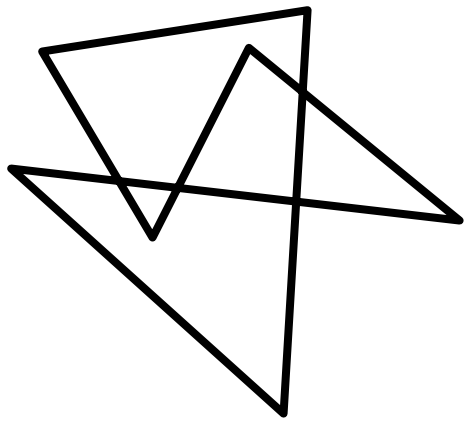
- Complex





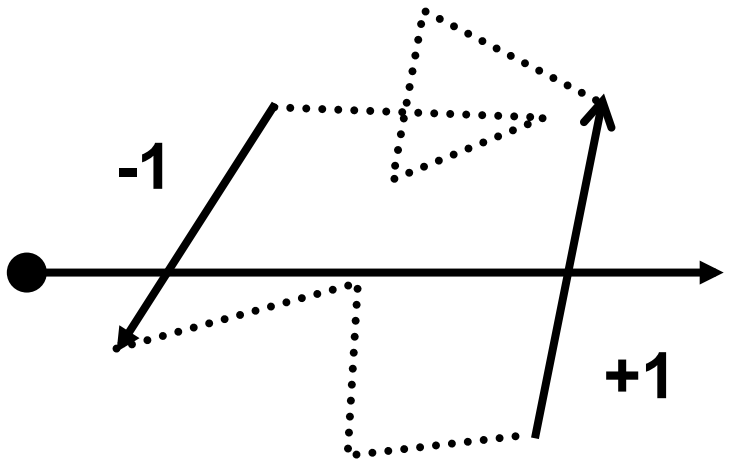
Inside and Outside

- Not just draw (stroke, fill)
- For closed Shapes
  - Hit test - inside or outside based on a winding rules (non-zero or even-odd)



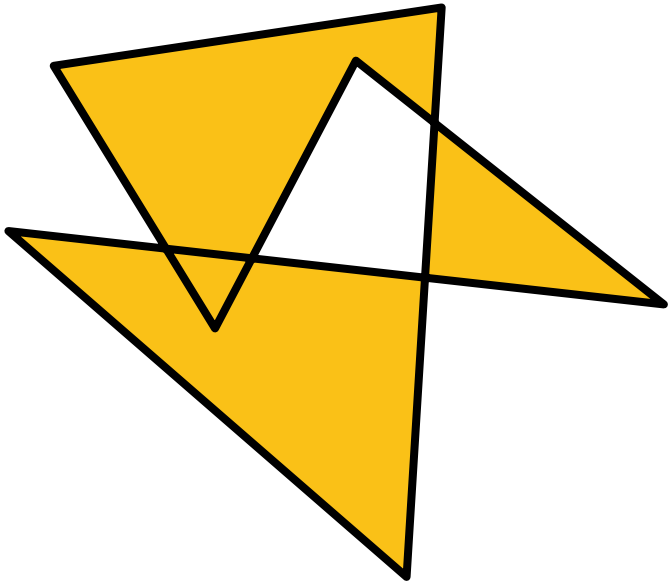
Counting Edge Crosses

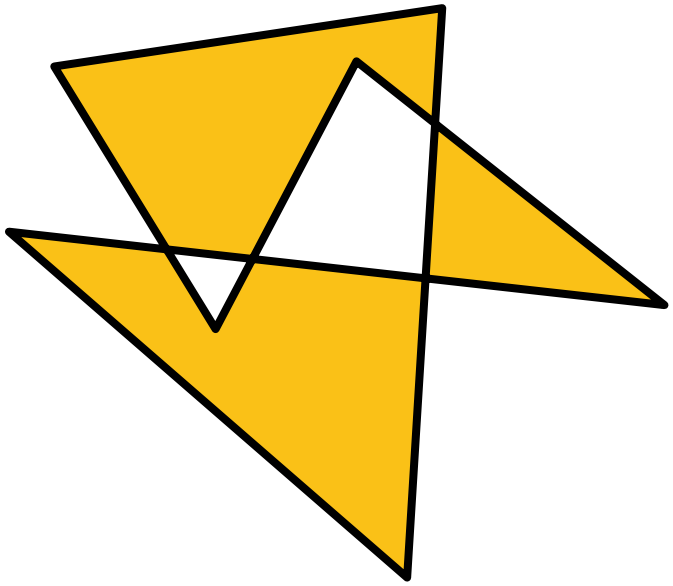
- Draw a line from the test point to the outside
  - Count +1 if you cross an edge in an anti-clockwise sense
  - Count -1 if you cross and edge in a clockwise sense



Winding Rules

- Non-zero
  - If total is non-zero then inside
- Odd-Even
  - If total is odd, then inside

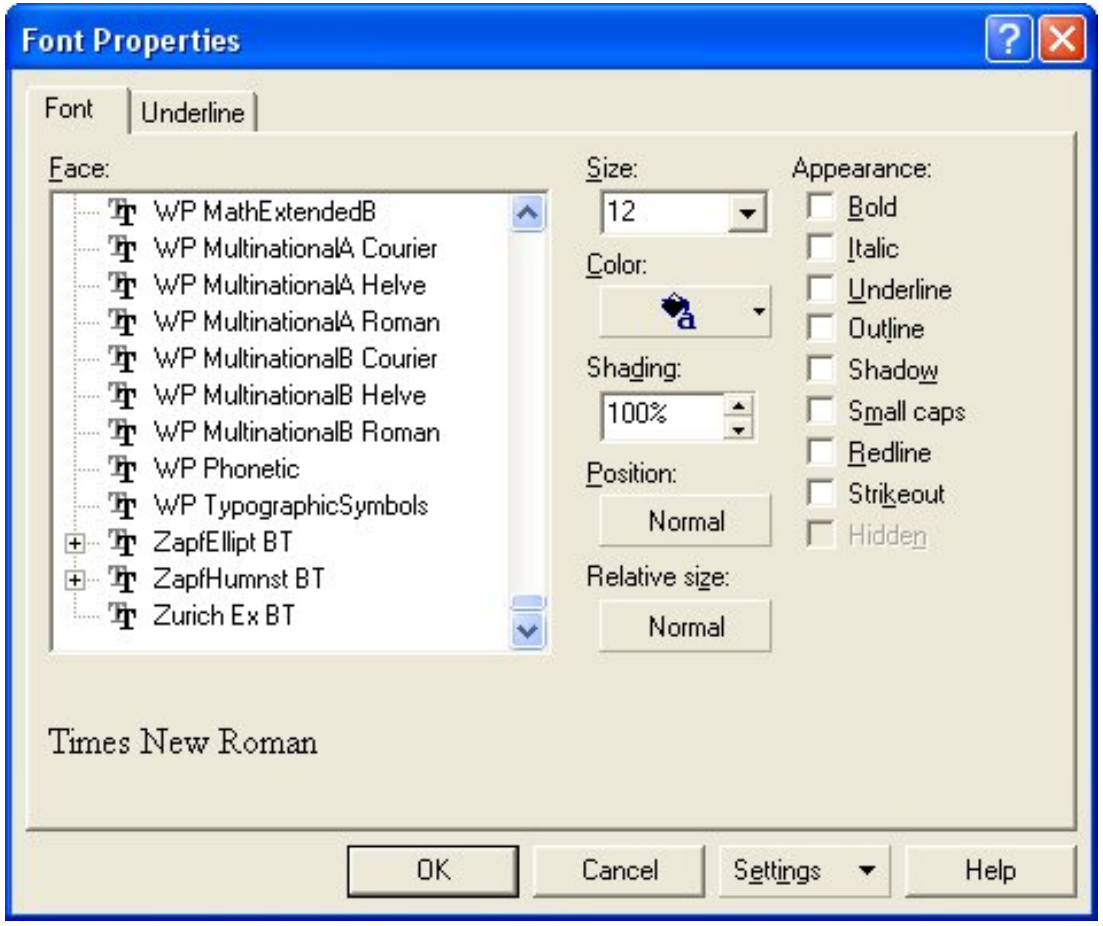




Fonts

- Font: describes the way a letter “looks”
- Size: usually specified in pt.
  - A pt is 1/72 of an inch
- Fixed width vs. proportional
  - Fixed width: each letter has fixed width
  - Proportional: width depends on letter

Font Properties



- Fonts can have different properties:
  - Bold
  - Italic
  - …

Representing Fonts

- Raster Fonts:
  - A bitmap for each letter
  - Scale dependent, i.e., need bitmaps for various sizes
- Vector Fonts:
  - Collection of line endpoints that define the letter.
  - Just lines!

Representing Fonts

- TrueType Fonts
  - Collection of lines and curves (Bezier) as well as hints.
  - Line and curves define outline of letter
  - Hints adjust lines and curves depending on scale of font!

- It's very tedious to create fonts!!

Summary

- Curve definitions
  - Bezier curves
- Inside and outside shapes
  - Winding rules
- Fonts